Chapter 1

Segal-Bargmann Space

In this chapter, we collect ideas and another perspective on the concentration of measure phenomenon. In particular, we look at symmetric products of $\mathbb{C}P^1$ and how they relate to Riemann surfaces and Segal-Bargmann spaces.

$$\mathbb{C}P^n \xleftrightarrow{\ w(z) \sim w(\lambda z)\ } \mathbb{P}^n$$

$$z \sim \lambda z \qquad w(z) = \sum_{i=0}^n Z_i z^i \qquad \text{root of } w(z)$$

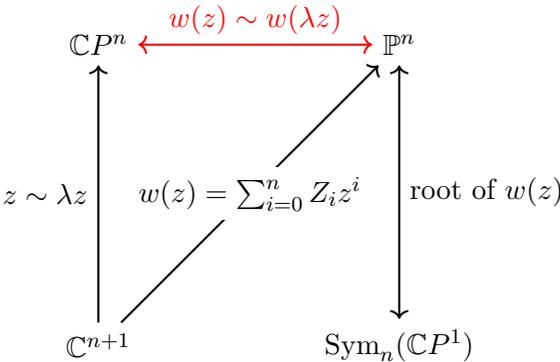$$\mathbb{C}^{n+1} \qquad\qquad \mathrm{Sym}_n(\mathbb{C}P^1)$$

Figure 1.1: Majorana stellar representation

Basically, there is a bijection between the complex projective space $\mathbb{C}P^n$ and the set of roots of a polynomial of degree n.

We can use a symmetric group of permutations of n complex numbers (or S^2) to represent the $\mathbb{C}P^n$, that is, $\mathbb{C}P^n = S^2 \times S^2 \times \cdots \times S^2/S_n$.

One might be interested in the random sampling over the $\mathrm{Sym}_n(\mathbb{C}P^1)$ and the concentration of measure phenomenon on that.

1.1 Majorana stellar representation of the quantum state

This branch continues the projective-space viewpoint from the previous chapters, but now through symmetric states and polynomial data. The advantage of this model is that it converts projective quantum states into configurations of points, making it plausible to compare concentration questions with geometry on symmetric products and holomorphic function spaces.

Definition 1. *Let n be a positive integer. The Majorana stellar representation of the quantum state is the set of all roots of a polynomial of degree n in $\mathbb{C}$.*

1.2   Space of complex valued functions and pure states