

Preface

This thesis studies how concentration of measure enters quantum information through the geometry of pure states. The basic question is probabilistic but the answer is geometric: once the state space is viewed as a high-dimensional metric-measure space, many physically relevant observables become sharply concentrated. In particular, the entropy of a random bipartite pure state is typically close to its maximal value, which explains why high-dimensional random states are generically highly entangled.

The exposition is organized in layers.

Chapter 0 collects the algebraic, probabilistic, geometric, and quantum-mechanical background needed later.

Chapter 1 develops the motivation from polarization experiments, recalls concentration on spheres, and then states the Hayden–Leung–Winter entanglement bound.

Chapter 2 reformulates the same phenomenon in the language of observable diameter and uses entropy-based simulations as a concrete probe on spheres and complex projective spaces.

Chapter 3 records an exploratory direction through Majorana stellar representation and related holomorphic models. It branches out from our main narrative but is included here for completeness and future reference as our journey into the geometry of quantum states continues.

How to use the dependency graph

The graph below is meant to be read from top to bottom. The first row lists undergraduate courses that supply the basic language. The middle rows list the sections and subsection-level themes that are used in the body of the thesis. The bottom node records the concentration-of-measure theorem that motivates the main narrative. Not every reader needs every path: a reader interested mainly in the entanglement theorem can follow the linear algebra, probability, quantum, and concentration branches first, while a reader interested in the geometric reformulation can continue through manifolds, Riemannian geometry, Hopf fibration, and observable diameter.

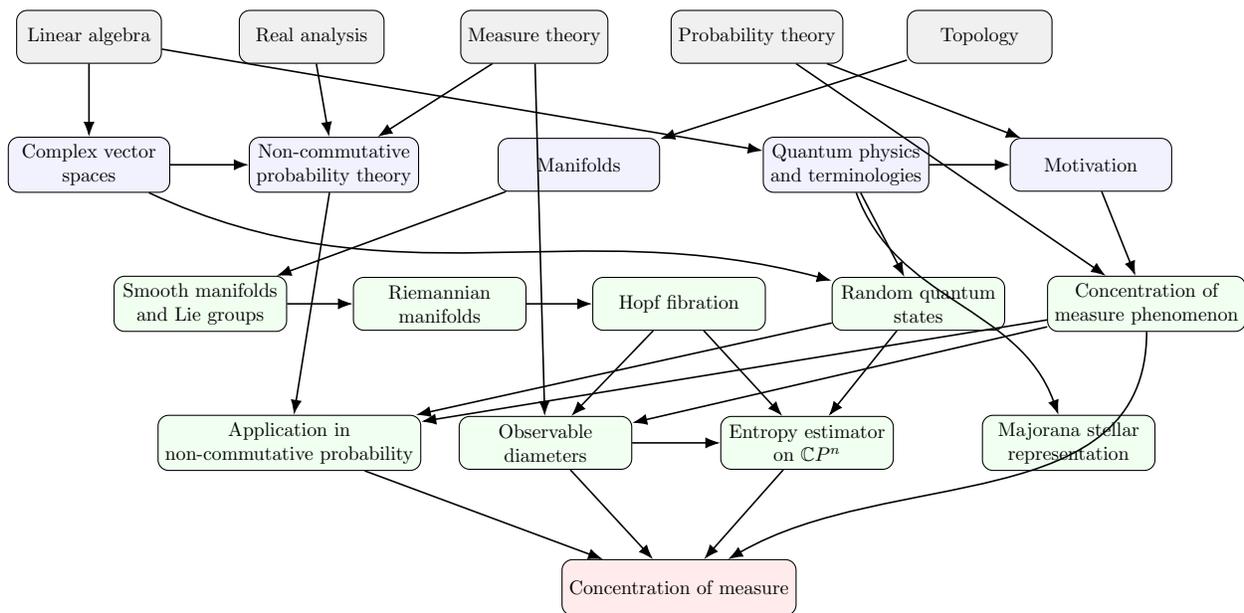


Figure 1: Dependency graph for the thesis. The central path runs from undergraduate background through geometry and quantum theory to concentration of measure, while the Majorana branch records a later exploratory direction.